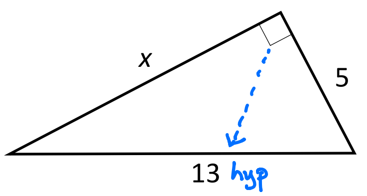
Application of Similar Right Triangles

STUDENT NOTES

Days 1-3	**2.4 <u>Right Triangle Relationships</u>** 16. I can solve for unknown sides using the 45°-45°-90° pattern 17. I can solve for unknown sides using the 30°-60°-90° pattern

Recall that the PYTHAGOREAN THEOREM shows that the relationship of the three sides of a RIGHT TRIANGLE is $a^2 + b^2 = c^2$ where a and b are the lengths of the legs and c is the length of the hypotenuse.

Ex 1: Solve for *x* in each. Your answer should ALWAYS be in simplified radical form.



$a^2+b^2=c^2$

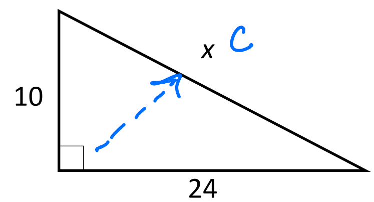
$5^2+x^2=13^2$

$25+x^2=169$

$-25 \qquad -25$

$\sqrt{x^2}=\sqrt{144}$

$\boxed{x=12}$



$\boxed{x=26}$

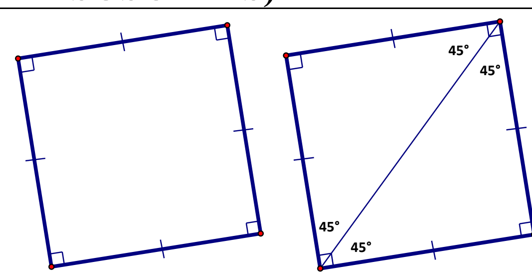
$10^2+24^2=x^2$

$100+576=x^2$

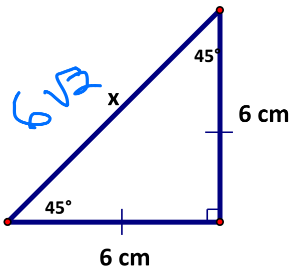
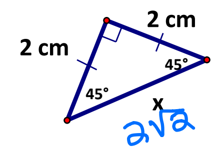
$\sqrt{676}=\sqrt{x^2}$

THE 45-45-90 TRIANGLE (RIGHT ISOSCELES)

The right isosceles is formed by drawing one of the diagonals of a square. The two triangles formed are congruent by SAS therefore the two parts of the right angles are congruent by CPCTC. This means the diagonal divides the 90° angle into two 45° angles. In addition, the legs of each triangle are congruent since all sides of a square are congruent.



UNDERSTANDING THE PATTERNS OF 45-45-90 TRIANGLES

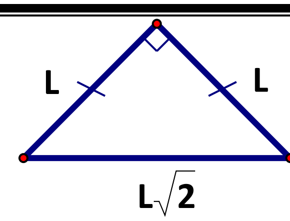
	
$a^2+b^2=c^2$ $6^2+6^2=x^2$ $36+36=x^2$ $72=x^2$ $\sqrt{72}=x$ $\sqrt{36}\bullet\sqrt{2}=x$ $6\sqrt{2}=x$	$a^2+b^2=c^2$ $2^2+2^2=x^2$ $4+4=x^2$ $8=x^2$ $\sqrt{8}=x$ $\sqrt{4}\bullet\sqrt{2}=x$ $2\sqrt{2}=x$

DO YOU SEE THE PATTERN? TRY THESE …

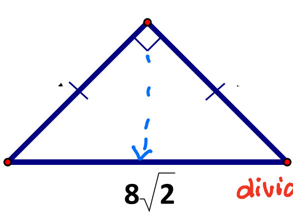
The legs are 4 and 4. What is the hypotenuse?	The legs are 10 and 10. What is the hypotenuse?	The legs are 36 and 36. What is the hypotenuse?
$4\cdot\sqrt{2}$ $\boxed{4\sqrt{2}}$	$10\cdot\sqrt{2}$ $\boxed{10\sqrt{2}}$	$36\cdot\sqrt{2}$ $\boxed{36\sqrt{2}}$

The legs of the right isosceles triangle are always congruent to each other because it is an isosceles triangle, and the hypotenuse will be $\sqrt{2}$ times longer than the legs. The ratio of the three sides of this triangle are $1 : 1 : \sqrt{2}$.

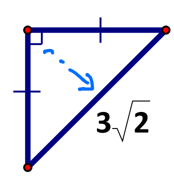
$$(\text{leg})\bullet(\sqrt{2})=\text{hypotenuse} \quad \text{or} \quad leg=\frac{hypotenuse}{\sqrt{2}}$$

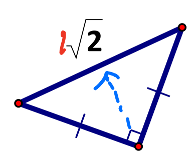


Knowing this relationship we can solve for the legs (which are congruent) of the triangle given the hypotenuse.



divide by √2

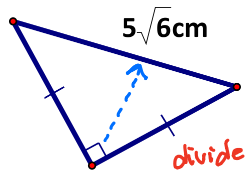




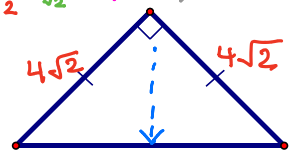
Leg	Leg	Hypotenuse	Leg	Leg	Hypotenuse	Leg	Leg	Hypotenuse
8	8	$8\sqrt{2}$	3	3	$3\sqrt{2}$	1	1	$\sqrt{2}$

$$\frac{8\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}} = 8\frac{\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$$

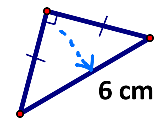
$\frac{5\sqrt{6}}{\sqrt{2}}$ $5\cdot\sqrt{\frac{\cancel{6}}{\cancel{2}}}\,3$



$\frac{8}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{8\sqrt{2}}{\sqrt{4}}=\frac{\overset{4}{\cancel{8}}\sqrt{2}}{\cancel{2}}$

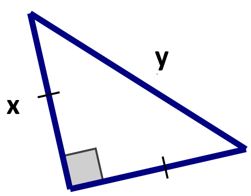


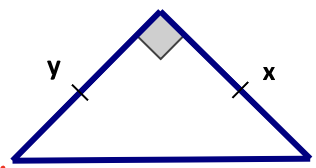
$\frac{6}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{6\sqrt{2}}{\sqrt{4}}=\frac{3\cancel{6}\sqrt{2}}{\cancel{2}}$



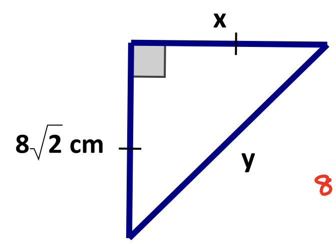
Leg	Leg	Hypotenuse	Leg	Leg	Hypotenuse	Leg	Leg	Hypotenuse
$5\sqrt{3}$	$5\sqrt{3}$	$5\sqrt{6}$	$4\sqrt{2}$	$4\sqrt{2}$	8	$3\sqrt{2}$	$3\sqrt{2}$	6

Now try these all mixed up. Solve for *x* and *y* in each.





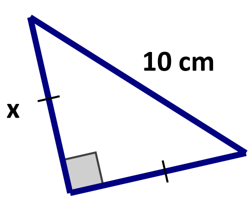
$\frac{15\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$

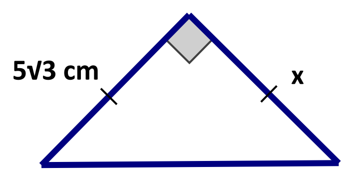


$8\sqrt{2}\cdot\sqrt{2}=$
$8\sqrt{4}=8\cdot 2$

$x =$ 5	$y =$ $5\sqrt{2}$	$x =$ 15	$y =$ 15	$x =$ $8\sqrt{2}$	$y =$ 16

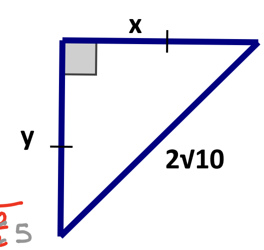
$\frac{10}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=$
$\frac{5\,\cancel{10}\sqrt{2}}{\cancel{2}}$





$5\sqrt{3}\cdot\sqrt{2}$

$\frac{2\sqrt{10}}{\sqrt{2}}$
$2\sqrt{\frac{\cancel{10}}{\cancel{2}}}\,5$

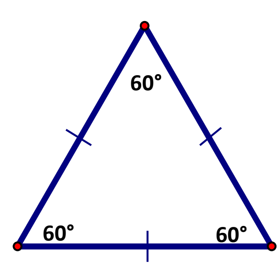


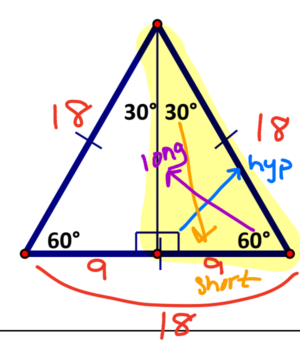
$x =$ $5\sqrt{2}$	$y =$ $5\sqrt{2}$	$x =$ $5\sqrt{3}$	$y =$ $5\sqrt{6}$	$x =$ $2\sqrt{5}$	$y =$ $2\sqrt{5}$

THE 30-60-90 TRIANGLE

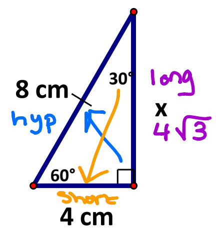
The 30-60-90 triangle is formed when the perpendicular bisector of a side of an equilateral triangle is drawn. This divides that side into two equal parts while also bisecting the opposite angle of 60° into two equal angles of 30°.

Since all three sides are congruent, the side that is bisected is exactly half of the hypotenuse. This relationship is easy to remember, so we can focus on the relationship these two sides have with the other leg.





UNDERSTANDING THE PATTERNS OF 30-60-90 TRIANGLES



$a^2 + b^2 = c^2$

$4^2 + x^2 = 8^2$

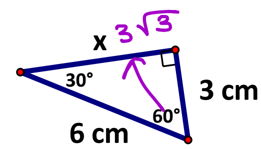
$16 + x^2 = 64$

$x^2 = 48$

$x = \sqrt{48}$

$x = \sqrt{16} \bullet \sqrt{3}$

$x = 4\sqrt{3}$



$a^2 + b^2 = c^2$

$3^2 + x^2 = 6^2$

$9 + x^2 = 36$

$x^2 = 27$

$x = \sqrt{27}$

$x = \sqrt{9} \bullet \sqrt{3}$

$x = 3\sqrt{3}$

DO YOU SEE THE PATTERN? TRY THESE …

The hypotenuse is 4. Find the shorter leg and longer leg.	The hypotenuse is 12. Find the shorter leg and longer leg.	The hypotenuse is 30. Find the shorter leg and longer leg.
shorter $= \frac{4}{2} = 2$	shorter $= \frac{12}{2} = 6$	Shorter $= \frac{30}{2} = 15$
longer $= 2 \cdot \sqrt{3} = 2\sqrt{3}$	longer $= 6 \cdot \sqrt{3} = 6\sqrt{3}$	longer $= 15 \cdot \sqrt{3} = 15\sqrt{3}$

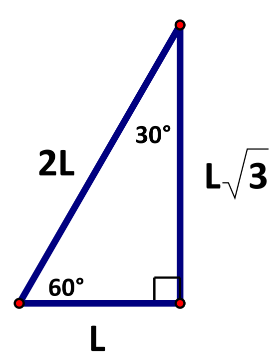
So we already know that the short leg is half of the hypotenuse.

Now we also know that the long leg is $\sqrt{3}$ times longer than the short leg.

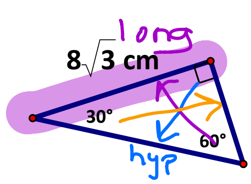
The ratio of the three sides of this triangle are $1 : \sqrt{3} : 2$.

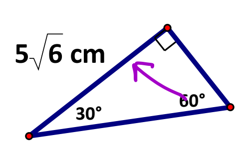
(short leg) $\bullet$ 2 = hypotenuse or $\text{short leg} = \dfrac{\text{hypotenuse}}{2}$

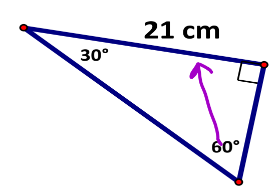
(short leg) $\bullet \sqrt{3}$ = long leg or $\text{short leg} = \dfrac{\text{long leg}}{\sqrt{3}}$



Knowing this relationship we can solve for the hypotenuse of the triangle given the longer leg .







Short Leg	Long Leg	Hypotenuse	Short Leg	Long Leg	Hypotenuse	Short Leg	Long Leg	Hypotenuse
8	$8\sqrt{3}$	16	$5\sqrt{2}$	$5\sqrt{6}$	$10\sqrt{2}$	$7\sqrt{3}$	21	$14\sqrt{3}$
					$2(5\sqrt{2})$	$\frac{21}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$		

$\frac{8\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}$

$2(8)$

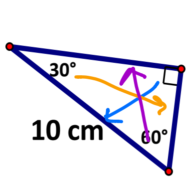
$\frac{5\sqrt{6}}{\sqrt{3}}$

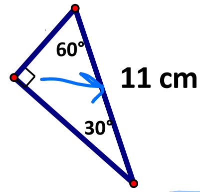
$5\sqrt{\frac{\cancel{6}\,2}{\cancel{3}}}$

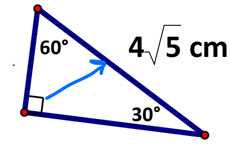
$\frac{7\,\cancel{21}\sqrt{3}}{\cancel{3}}$

$2(7\sqrt{3})$

And of course we can solve for the legs given the hypotenuse.

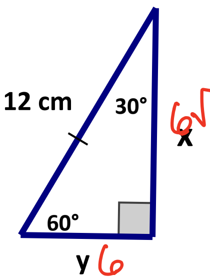


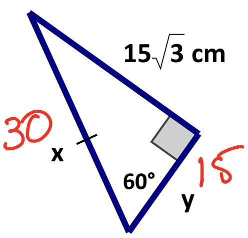


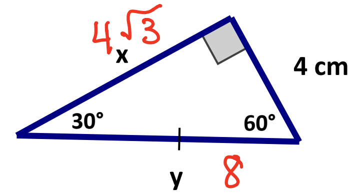


Short Leg	Long Leg	Hypotenuse	Short Leg	Long Leg	Hypotenuse	Short Leg	Long Leg	Hypotenuse
5	$5\sqrt{3}$	10	5.5	$5.5\sqrt{3}$	11	$2\sqrt{5}$	$2\sqrt{15}$	$4\sqrt{5}$
$\frac{10}{2}$	$5\sqrt{3}$		$\frac{11}{2}$	$5.5\sqrt{3}$		$\frac{4\sqrt{5}}{2}$	$2\sqrt{5}\cdot\sqrt{3}$	

Now try these all mixed up. Solve for *x* and *y* in each.

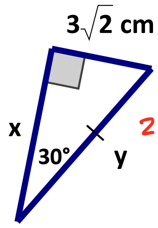




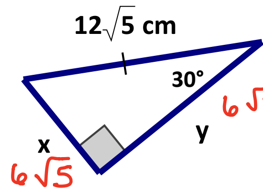


$x =$ $6\sqrt{3}$	$y =$ 6	$x =$ 30	$y =$ 15	$x =$ $4\sqrt{3}$	$y =$ 8

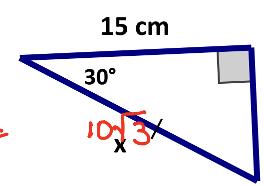
$3\sqrt{2}\cdot\sqrt{3}$ $\quad 2\cdot 3\sqrt{2}$



$\frac{\cancel{12}\sqrt{5}}{\cancel{2}}$ $\quad 6\sqrt{5}\cdot\sqrt{3}$



$2\cdot 5\sqrt{5}$ $\quad \frac{15}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{15\sqrt{3}}{3}$



$x =$ $3\sqrt{6}$	$y =$ $6\sqrt{2}$	$x =$ $6\sqrt{5}$	$y =$ $6\sqrt{15}$	$x =$ $10\sqrt{3}$	$y =$ $5\sqrt{3}$

Day 5	Quiz 2.4 – Targets 15-17

Day 6	2.5 **Right Triangle Trigonometry** 18. I can label a right triangle in relation to the reference angle (opposite, adjacent & hypotenuse).

SINE — reference angle — $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

COSINE — reference angle — $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

TANGENT — $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ — reference angle

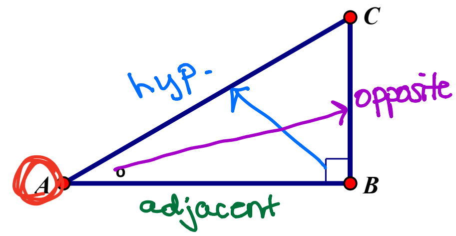
Learning to **Label a Side** based off a **Reference Angle**.

Hypotenuse -- Longest Side -- Always Opposite the Right Angle.

Opposite Leg -- Leg of the triangle that does not form the reference angle.

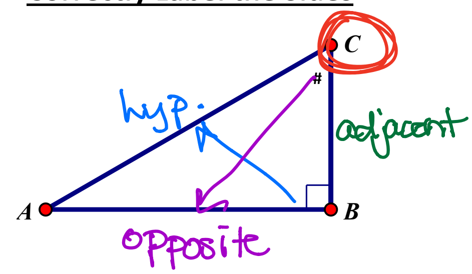
Adjacent Leg -- Non-Hypotenuse leg of the reference angle.

Correctly Label the sides.



Reference Angle is ∠A

Correctly Label the Sides



Reference Angle is ∠C

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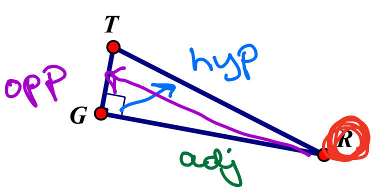
1. Determine the correct side based on the reference angle.

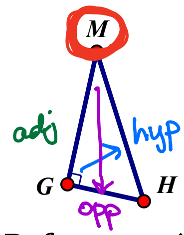
a)



Reference ∠R

Opposite Side	$\overline{GT}$
Adjacent Side	$\overline{GR}$
Hypotenuse	$\overline{RT}$

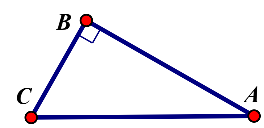
b)



Reference ∠M

Opposite Side	$\overline{GH}$
Adjacent Side	$\overline{GM}$
Hypotenuse	$\overline{MH}$

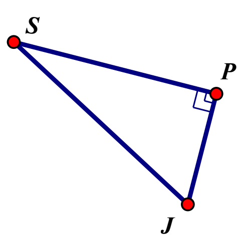
c)



Reference ∠C

Opposite Side	$\overline{AB}$
Adjacent Side	$\overline{BC}$
Hypotenuse	$\overline{AC}$

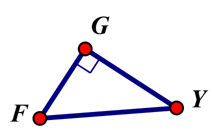
d)



Reference ∠S

Opposite Side	$\overline{PJ}$
Adjacent Side	$\overline{SP}$
Hypotenuse	$\overline{SJ}$

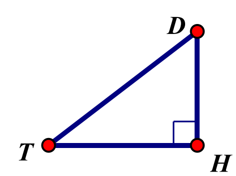
e)



Reference ∠Y

Opposite Side	$\overline{FG}$
Adjacent Side	$\overline{GY}$
Hypotenuse	$\overline{YF}$

f)



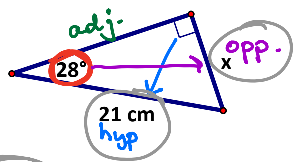
Reference ∠D

Opposite Side	$\overline{TH}$
Adjacent Side	$\overline{DH}$
Hypotenuse	$\overline{DT}$

2. Label the sides based on the triangle using the reference angle.

- (O) for Opposite, (A) for Adjacent and (H) for Hypotenuse.
- After you have labeled the triangle, then choose which trigonometric ratio that you would use to solve for the missing info.

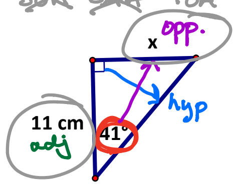
a) SOH-~~CAH~~-~~TOA~~



SIN COS TAN

$\sin 28^\circ = \frac{x}{21}$

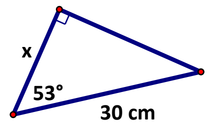
b) ~~SOH~~-~~CAH~~-TOA



SIN COS TAN

$\tan 41^\circ = \frac{x}{11}$

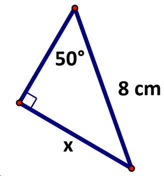
c)



SIN COS TAN

$\cos 53^\circ = \frac{x}{30}$

d)

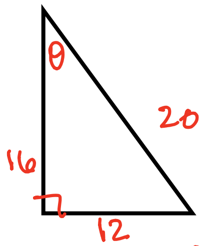


SIN COS TAN

$\sin 50^\circ = \frac{x}{8}$

3. Find the trig ratios (sine, cosine, tangent) using smallest angle of a right triangle as the refence angle with the given lengths:

a) 12, 16, 20

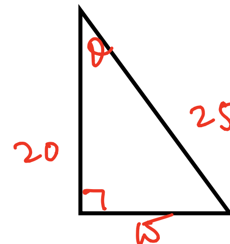


sin $\theta =$ $\frac{12}{20} = \frac{3}{5}$

cos $\theta =$ $\frac{16}{20} = \frac{4}{5}$

tan $\theta =$ $\frac{12}{16} = \frac{3}{4}$

b) 15, 20, 25



sin $\theta =$ $\frac{15}{25} = \frac{3}{5}$

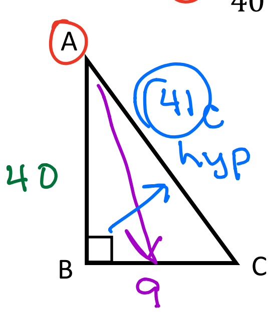
cos $\theta =$ $\frac{20}{25} = \frac{4}{5}$

tan $\theta =$ $\frac{15}{20} = \frac{3}{4}$

What do you notice about the ratios of the two triangles above? Why did this happen?

$\frac{\text{opp}}{\text{adj}}$ $\frac{9}{40}$

4. Given $\tan A = \frac{9}{40}$, find $\sin A$.



$\sin A = \frac{\text{opp}}{\text{hyp}} = \boxed{\frac{9}{41}}$

$a^2 + b^2 = c^2$

$9^2 + 40^2 = c^2$

$81 + 1600 = c^2$

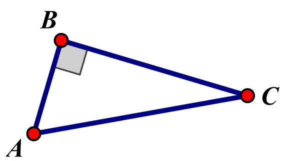
$\sqrt{1681} = \sqrt{c^2}$

$c = 41$

Days 7-8	**2.5 Right Triangle Trigonometry** 18. I can label a right triangle in relation to the reference angle (opposite, adjacent & hypotenuse). 19. I can use trigonometry to solve for sides and angles of right triangles.

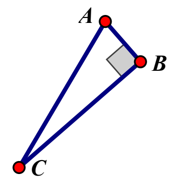
Warm-Up: Identify the following:

1. What is the side opposite ∠A?



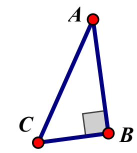
$\overline{BC}$

2. What is the hypotenuse?



$\overline{AC}$

3. What is the adjacent side to ∠C?

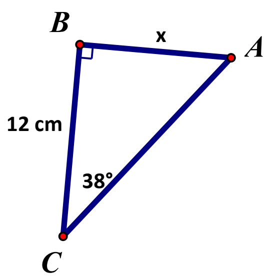


$\overline{BC}$

Sine = $\frac{\text{Opposite}}{\text{Hypotenuse}}$ Cosine = $\frac{\text{Adjacent}}{\text{Hypotenuse}}$ Tangent = $\frac{\text{Opposite}}{\text{Adjacent}}$

Using Trigonometry to determine the missing side.

a)



$$\tan\theta = \frac{Opposite}{Adjacent}$$

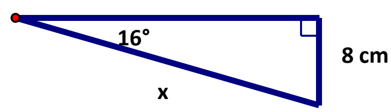
$$\tan 38° = \frac{x}{12}$$

$$\frac{\tan 38°}{1} = \frac{x}{12}$$

$$x = 12 \bullet \tan 38°$$

$$x \approx 9.38\ cm$$

b)



$$\sin\theta = \frac{Opposite}{Hypotenuse}$$

$$\sin 16° = \frac{8}{x}$$

$$\frac{\sin 16°}{1} = \frac{8}{x}$$

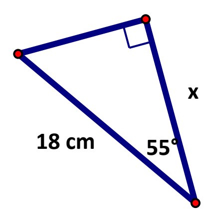
$$x \bullet \sin 16° = 8$$

$$\frac{x \bullet \cancel{\sin 16°}}{\cancel{\sin 16°}} = \frac{8}{\sin 16°}$$

$$x = \frac{8}{\sin 16°}$$

$$x \approx 29.02\ cm$$

c)



$$\cos\theta = \frac{Adjacent}{Hypotenuse}$$

$$\cos 55° = \frac{x}{18}$$

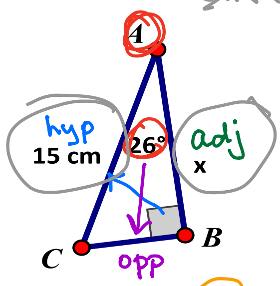
$$\frac{\cos 55°}{1} = \frac{x}{18}$$

$$x = 18 \bullet \cos 55°$$

$$x \approx 10.32\ cm$$

Find the missing side length for each problem below. Be sure to write the trig. ratio set-up you are using to solve each problem.

1.

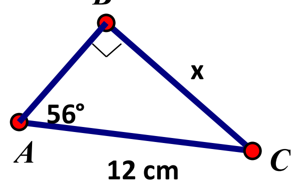


$\cos 26° = \frac{x}{15}$

$x = 15 \cdot \cos 26°$

$x \approx$ 13.48cm

2.



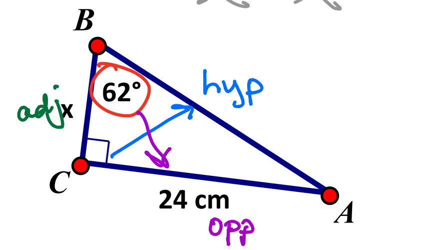
$\sin 56° = \frac{x}{12}$

$x = 12 \cdot \sin 56°$

$x \approx$ 9.95cm

3.

$S\frac{O}{H}$ $C\frac{A}{H}$ $T\frac{O}{A}$



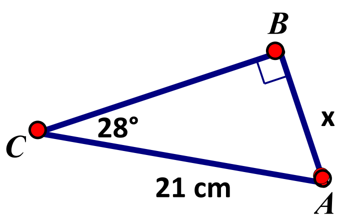
$\tan 62° = \frac{24}{x}$

$\frac{24}{\tan 62°} = \frac{x \cdot \cancel{\tan 62°}}{\cancel{\tan 62°}}$

$x = \frac{24}{\tan 62°}$

$x \approx$ 12.76 cm

4.

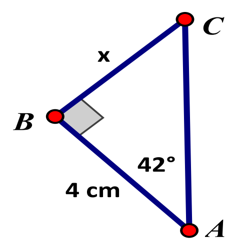


$\sin 28° = \frac{x}{21}$

$x = 21 \cdot \sin 28°$

$x \approx$ 9.86 cm

5.

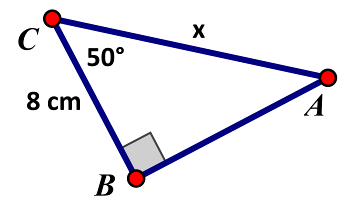


$\tan 42° = \frac{x}{4}$

$x = 4 \cdot \tan 42°$

$x \approx$ 3.60 cm

6.



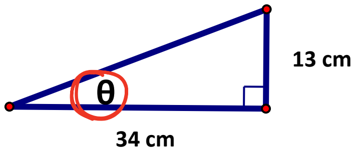
$\cos 50° = \frac{8}{x}$

$x = \frac{8}{\cos 50°}$

$x \approx$ 12.45 cm

Using Trigonometry to determine the missing angle.

a)



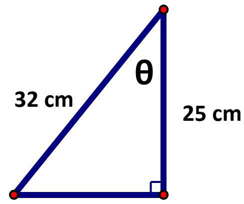
$$\tan\theta = \frac{Opposite}{Adjacent}$$

$$\cancel{\tan^{-1}}(\tan\theta) = \cancel{\frac{13}{34}}\ \tan^{-1}\left(\frac{13}{34}\right)$$

$$\theta \approx \tan^{-1}\left(\frac{13}{34}\right)$$

$$\theta \approx 20.92°$$

b)



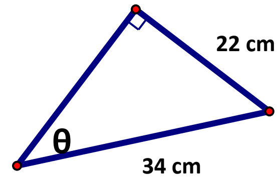
$$\cos\theta = \frac{Adjacent}{Hypotenuse}$$

$$\cos^{-1}(\cos\theta) = \cancel{\frac{25}{32}}\ \cos^{-1}\left(\frac{25}{32}\right)$$

$$\theta \approx \cos^{-1}\left(\frac{25}{32}\right)$$

$$\theta \approx 38.62°$$

c)



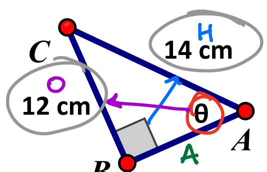
$$\sin\theta = \frac{Opposite}{Hypotenuse}$$

$$\sin\theta = \frac{22}{34}$$

$$\theta \approx \sin^{-1}\left(\frac{22}{34}\right)$$

$$\theta \approx 40.32°$$

7.



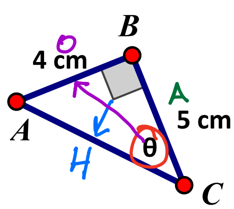
SOH ~~CAH~~ ~~TOA~~

$\sin\theta = \frac{12}{14}$

$\theta = \sin^{-1}\left(\frac{12}{14}\right)$

$\boxed{\theta \approx 59°}$

8.

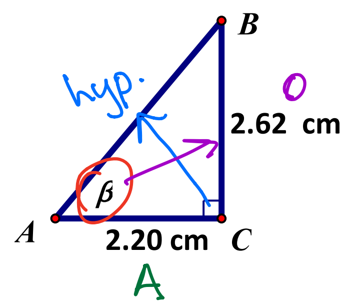


$\tan\theta = \frac{4}{5}$

$\theta = \tan^{-1}\left(\frac{4}{5}\right)$

$\boxed{\theta \approx 38.66°}$

9.



$\tan\beta = \frac{2.62}{2.20}$

$\beta = \tan^{-1}\left(\frac{2.62}{2.20}\right)$

$\boxed{\beta \approx 50°}$

$\theta \approx$ ______________

$\theta \approx$ ______________

$\beta \approx$ ______________

SOH - CAH - TOA

S	O	H	C	A	H	T	O	A
Sine	Opposite	Hypotenuse	Cosine	Adjacent	Hypotenuse	Tangent	Opposite	Adjacent

$S\ \frac{O}{H} \qquad C\ \frac{A}{H} \qquad T\ \frac{O}{A}$